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Gas Network Simulation with the Aid of the Method of Characteristics *

1. Introduction

At the dawn of new millenium the natural gas economy in Germany faces quite new challenges. The EU internal market, the new frame conditions for policy in the field of power distribution, the opening and liberalism of power markets and the sharpening of gas-to-gas competition associated with it urge to a higher flexibility, to the introduction of new marketing concepts and to a higher gaining of the gas delivery enterprises via the use of new or not yet exhausted potentials for cost savings.

In view of the new market conditions, the gas network simulation, that is obtaining the data approximating the information about the nonstationary flow processes in the regional and superregional high-pressure networks, wins an increasing importance. The temporal separation of amount and price, the control of very various properties of natural gas, as well as storage and effective use of **Spottmengen** lead to quite new tasks, which are related to a high economical hazard and can be solved rapidly only with the aid of qualitatively new control technology and the most advanced simulation techniques.

In this article, a mathematical model is presented for the computation of highly dynamical and, in particular, non-isothermal flow processes in gas networks on the basis of the knowledge and experience in the field of gas network simulation accompanying the process. The basis of this model is constituted by the method of characteristics, which has still not found any widespread application in Germany despite its outstanding properties.

The method of characteristics represents an innovative supplement to such well-known simulation techniques verified in practice as GANESI and SIMONE. This method will win the practical importance for the simulation of highly dynamical flow processes, optimization of pre-heating, control of gas properties, obtaining the new knowledge in the field of retrograde condensation as well as for the modelling of multiphase flows.

1. Developments in the Field of Gas Network Simulations

The first works in the field of computation of nonstationary, i.e. time dependent flow processes in pipelines were carried out already in the early 20th century by Joukowksy [1] and Alliévi [2]. The analytical methods enabled then one to solve and to represent graphically simple problems.

The development of the first computational methods for nonstationary flows in gas pipelines, that is for the computation of time-variable pressure and flux distribution in the network under variable boundary conditions had begun in the early sixties of the 20th century.

At that time, the computations were else very time-consuming and required the use of a big computer. At present, such computations are carried out preferably with the aid of numerical solution techniques on small but very powerful desktop computers. A breakthrough in the nonstationary simulation of flow processes in gas networks was made in 1976 by Weimann [3] with his code GANESI. This code or its specific variants for different firms are now used in many European countries [4].

Besides GANESI, which is exploited since 2000 by PSI AG, there are a number of other codes for nonstationary simulation of flow processes in gas networks. These are, in particular:

- the code developed by SIMONE Research Group in Prague, Czech Republic, which is used in Germany by the LIWACOM firm [6];
- the USA codes
  - of the Enterprise Group STONER Associates Inc., the STONER - PIPELINE - Simulator (SPS) [7] and

- of the Gregg Engineering, the code WinTran™Online™ [8];
- the Canadian code PIPEFLOW of the Neotechnology Consultants Ltd. [9];
- the Australian code FlowTran of the firm William J. Turner Pty Ltd. [10]
- and the Danish Gas Network-Management-Software of the firm LIC*ENERGY* [11].

There are the interested groups, which are concerned with the exchange of new knowledge and experience in the field of gas network simulation, for the code GAMOS (it incorporates the computational kernel of GANESI), SIMONE, and for a number of other codes from USA, Canada, Australia, Europe, and Asia. The Pipeline Simulation Interest Group with headquarters in the USA [12] is the most well known.

2. Fundamentals of Gas Network Simulation

3.1 Derivation of Model Equations

The conservation laws for the mass, momentum, and energy known from the gas and fluid dynamics [13,14,15] constitute the basis for the description of a one-dimensional nonstationary gas flow in a pipeline:

- continuity equation

$$\frac{d}{d\,t}\int_V \rho\, d\,\tau = 0\ ; \tag{1}$$

- momentum equation

$$\frac{d}{d\,t}\int_V \rho\cdot\omega\, d\,\tau = \oint_\Omega P_n\, d\sigma + \int_V \rho\cdot F\, d\,\tau\ ; \tag{2}$$

- energy equation

$$\frac{d}{d\,t}\int_V \rho\left(\varepsilon+\frac{\omega^2}{2}\right) d\,\tau = \oint_\Omega P_n\cdot\omega\, d\sigma + \int_V \rho\, F\cdot\omega\, d\,\tau + \oint_\Omega q_n\, d\sigma\ ; \tag{3}$$

After the conversion of surface integrals to volume integrals (using the Gauss integral theorem), differentiation of these volume integrals with respect to time, transition to limit V → 0, and the substitution of the values of gas density and velocity averaged over the pipe cross-section according to formulas

$$A(x,t) = \langle A(x,y,z,t) \rangle = \frac{4}{\pi \cdot D^2} \int A(x,y,z,t)\ d\sigma\ ; \qquad (4)$$

the system of partial differential equations follows from the above equations for the computation of four unknown functions ρ **(x,t), p (x,t),** $\dot{m}$ **(x,t) and T(x,t)** :

$$\frac{\partial\ \rho}{\partial\ t} + \frac{\partial\ \dot{m}}{\partial\ x} = 0\ ; \qquad (5)$$

$$\frac{\partial\ \dot{m}}{\partial\ t} + \frac{\partial\ p}{\partial\ x} = -\frac{\lambda}{2 \cdot D} \cdot \frac{\dot{m} \cdot |\dot{m}|}{\rho}\ ; \qquad (6)$$

$$\rho \cdot c_p\ \frac{\partial\ T}{\partial\ t} + \dot{m} \cdot c_p\ \frac{\partial\ T}{\partial\ x} - \frac{\partial\ p}{\partial\ t} = \frac{4 \cdot q_n}{D}\ ; \qquad (7)$$

$$p = \rho\ R\ T\ . \qquad (8)$$

Equations (5) to (8) form a system of nonlinear partial differential equations of hyperbolic type. It is characterized by a finite speed of the propagation of disturbances, which agrees very well with the gas adiabatic speed of sound.

The gas temperature in a pipeline generally differs little from its immediate environment. The derivative $\partial\ T / \partial\ x$ and the heat flux 4 q / D can, therefore, be neglected. The following relation then holds for temperature oscillation:

$$\frac{\partial\ T}{\partial\ t} = \frac{1}{\rho \cdot c_p} \frac{\partial\ p}{\partial\ t}\ . \qquad (9)$$

Strictly speaking the nonstationary flow processes in gas pipelines cannot be considered as the isothermal processes. Right because the heat exchange in gas proceeds very slowly the pressure and mass flow rate oscillations are always accompanied by a local temperature change. The heat released thereat cannot be transferred sufficiently quickly from the gas particles, that is the process occurs practically under the adiabatic conditions [16].

If we extract the density in the continuity equation with the aid of the equation of state

$$\frac{\partial p}{\partial t} - \rho \cdot R \frac{\partial T}{\partial t} + R \cdot T \frac{\partial \dot{m}}{\partial x} = 0 \; ; \qquad (10)$$

and replace the derivative of temperature with respect to time by equation (9)

$$\frac{\partial p}{\partial t} \cdot \left(1 - \frac{R}{c_p}\right) + R \cdot T \frac{\partial \dot{m}}{\partial x} = 0 \; ; \qquad (11)$$

then we obtain with

$$R = c_p - c_v \; , \qquad \gamma = c_p / c_v \; , \qquad c^2 = \left(\frac{d p}{d \rho}\right)_s = \gamma \cdot R \cdot T \qquad (12)$$

the well-known system of equations

$$\frac{\partial p}{\partial t} + c^2 \frac{\partial \dot{m}}{\partial x} = 0 \; ;$$

$$(13)$$

$$\frac{\partial \dot{m}}{\partial t} + \frac{\partial p}{\partial x} = -\frac{\lambda \cdot c^2}{2 \cdot \gamma \cdot D} \cdot \frac{\dot{m}\left|\dot{m}\right|}{p} \; ;$$

for the computation of two functions $\boldsymbol{p}$(x, t) and $\dot{\boldsymbol{m}}$(x, t) of a nonstationary pipe flow under isothermal conditions.

The system of equations (13) constitutes, in this or slightly different form, the basis of the mathematical model for a variety of codes for gas network simulation [17, 18, 19, 20].

3.2 Analytic Solution Methods

In the 60ies and 70ies years of the 20th century there were numerous attempts at solving the system of nonlinear partial differential equations (13) analytically. The purpose was to obtain with the aid of the computer and measuring technology, which was available at that time, as actual as possible assertions about the time dependent flow processes in a high-pressure network, which is an important prerequisite for its rapid observation and control.

Although the numerous works [21, 22, 23, 24] in this field of the mathematical physics have not gained any widespread acceptance in the gas network simulation, they are nevertheless of significant theoretical and practical importance for the understanding of the physics of nonstationary flows.

An important preliminary operation for the analytic solution of the system of differential equations (13) is its linearization. The techniques, in which the so-called friction term is defined as follows, have gained a widespread acceptance:

$$\mathbf{2}\boldsymbol{a} = \frac{\boldsymbol{\lambda} \cdot \overline{\boldsymbol{\omega}}}{\mathbf{2} \cdot \boldsymbol{D}} \quad . \tag{15}$$

With this ansatz, the system of equations (13) simplifies as follows:

$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{t}} + \boldsymbol{c}^2 \frac{\partial \dot{\boldsymbol{m}}}{\partial \boldsymbol{x}} = \mathbf{0} \ ;$$

$$\frac{\partial \dot{\boldsymbol{m}}}{\partial \boldsymbol{t}} + \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{x}} = -\mathbf{2}\boldsymbol{a} \cdot \dot{\boldsymbol{m}} \quad . \tag{16}$$

The both equations can be reverted to a single second-order partial differential equation (the telegraph equation) both for the pressure an for the mass flow rate:

$$c^2 \; \frac{\partial^2 \; p}{\partial \; x^2} = \frac{\partial^2 \; p}{\partial \; t^2} + 2a \cdot \frac{\partial \; p}{\partial \; t} \quad ;$$

(17)

$$c^2 \; \frac{\partial^2 \; \dot{m}}{\partial \; x^2} = \frac{\partial^2 \; \dot{m}}{\partial \; t^2} + 2a \cdot \frac{\partial \; \dot{m}}{\partial \; t} \quad .$$

From these equations it follows for the so-called „long pipe model“ that the inertia forces are much smaller than the friction losses and can, therefore, be neglected, which leads to the heat conduction equation

$$c^2 \; \frac{\partial^2 \; p}{\partial \; x^2} = 2a \cdot \frac{\partial \; p}{\partial \; t} \quad ;$$

(18)

$$c^2 \; \frac{\partial^2 \; \dot{m}}{\partial \; x^2} = 2a \cdot \frac{\partial \; \dot{m}}{\partial \; t} \quad ;$$

or, for the so-called „short pipe model“, that the friction forces are much smaller than the inertia forces and, therefore, can be neglected, which leads to the wave equation:

$$c^2 \; \frac{\partial^2 \; p}{\partial \; x^2} = \frac{\partial^2 \; p}{\partial \; t^2} \quad ;$$

(19)

$$c^2 \; \frac{\partial^2 \; \dot{m}}{\partial \; x^2} = \frac{\partial^2 \; \dot{m}}{\partial \; t^2} \quad ;$$

The analytic solution of such second-order partial differential equations for simple initial and boundary conditions represents no problem when using the Fourier or Laplace transform. The "Symbolic Mathematics Software" (Macsyma, Maple, Mathematica) is now used more and more frequently for the solution of suh tasks.

Example:

The pressure curve in a gas pipeline with length **L** and diameter **D** before and after gas release to the moment of time **$t = t_1$** in the amount of $\dot{m}_A$ at the pipeline point **$x = x_1$** is to be computed. The general solution of the following partial differential equation is sought for:

$$\frac{\partial^2 p}{\partial x^2} = \frac{2a}{c^2} \cdot \frac{\partial p}{\partial t} + \frac{2a \cdot \dot{m}_A}{A} \cdot \left[\sigma(t) - \sigma(t - t_1)\right] \delta(x - x_1); \qquad \textbf{(20)}$$

which satisfies the following initial and boundary conditions:

$$p_{x=0} = p_a \; , \qquad p_{x=L} = p_e \; , \qquad p_{t=0} = p_0 \; . \qquad \textbf{(21)}$$

The general solution computed with the aid of the software system "Macsyma" [25] is

$$p(x,t) = p_a - \frac{p_a - p_e}{L} \cdot x + \sum_{k=1}^{\infty} \Bigg(\frac{2 \cdot p_0}{\pi \cdot k} \cdot \left[(-1)^{k-1} + 1\right] - \frac{2 \cdot \left[(-1)^{k-1} \cdot p_e + p_a\right]}{\pi \cdot k} +$$

$$+ \frac{2 \cdot 2a \cdot \dot{m}_A \cdot L}{A \cdot (k \cdot \pi)^2} \cdot \sin\left(\frac{k \cdot \pi \cdot x_1}{L}\right) \Bigg) \cdot \exp\left[-\frac{c^2}{2a} \left(\frac{k \cdot \pi}{L}\right)^2 \cdot L \right] \cdot \sin\left(\frac{k \cdot \pi \cdot x}{L}\right) - \qquad \textbf{(22)}$$

$$+ \frac{2 \cdot 2a \cdot \dot{m}_A \cdot L}{A \cdot \pi^2} \cdot \sum_{k=1}^{\infty} \frac{\sin\left(\frac{k \cdot \pi \cdot x_1}{L}\right)}{k^2} \cdot \exp\left[-\frac{c^2}{2a} \left(\frac{k \cdot \pi}{L}\right)^2 \cdot (t - t_1) \right] \cdot \sigma(t - t_1)$$

$$\cdot \sin\left(\frac{k \cdot \pi \cdot x}{L}\right) - \left[\sigma(t) - \sigma(t - t_1)\right] \frac{2a \cdot \dot{m}_A}{A} \cdot \begin{cases} \dfrac{x \cdot (L - x_1)}{L} & \text{for } x < x_1 \end{cases} .$$

$$\frac{x_1 \cdot (L - x)}{L} \; bei \;\; x > x_1$$

Intense research is being conducted in the field of symbolic solution of partial differential equations in USA and Canada [26, 27].

3.2 Numerical Solution

The variety of computer codes available for the gas network simulation are based on an iterative numerical integration of the system of partial differential equations (13).

For relatively slow (quasistationary) duct flows, the numerical method developed by Weimann [3, 18] and employed in the software package GANESI (**GA**S **NE**TWORK **SI**MULATION PROGRAM) has proved to be remarkable in practice.

In this method, the system of partial differential equations retaining the time derivatives is reverted into a semidiscrete system of ordinary differential equations. The resulting substitution model (Figure 1) has the structure of a chain conductor with concentrated parametric elements storage (the state quantity **p**), the friction drag and mass inertia (assigned jointly to the state quantity $\dot{m}$), which includes the boundary conditions.

The differential equations of state read [28]:

$$\frac{\partial p_1}{\partial t} = - \frac{\alpha}{\Delta x} \cdot (\dot{m}_2 - \dot{m}_e(t))$$

$$\frac{\partial \dot{m}_2}{\partial t} = - \frac{\beta}{2 \cdot \Delta x} \cdot (p_3 - p_1) - 2 \cdot \chi \cdot \frac{|\dot{m}_2| \cdot \dot{m}_2}{p_3 + p_1}$$

$$\frac{\partial p_3}{\partial t} = - \frac{\alpha}{\Delta x} \cdot (\dot{m}_4 - \dot{m}_2)$$

$$\frac{\partial \dot{m}_n}{\partial t} = - \frac{\beta}{2 \cdot \Delta x} \cdot (p_{n+1} - p_{n-1}) - 2 \cdot \chi \cdot \frac{|\dot{m}_n| \cdot \dot{m}_n}{p_{n+1} + p_{n-1}}$$

$$\frac{\partial\ p_{n+1}}{\partial\ t} = -\ \frac{\alpha}{\Delta\ x} \cdot (-\dot{m}_n + \dot{m}_a(t)) \tag{23}$$

If the state variables **p** and $\dot{m}$ are considered as the components of state vector $\underline{y}$, then the concentrated parametric substitution model may be written also as a vector differential equation:

$$\frac{\partial\ \underline{y}}{\partial\ t} = \underline{f}(\underline{y}) + B \cdot \underline{u} \tag{24}$$

With the state vector

$$\underline{y} = \begin{bmatrix} p_1 & \dot{m}_2 & p_3 & \ldots & \dot{m}_n & p_{n+1} \end{bmatrix}^T \tag{25}$$

and the boundary value vector

$$\underline{u} = \begin{bmatrix} \dot{m}_e(t) & \dot{m}_a(t) \end{bmatrix}^T . \tag{26}$$

A number of important criteria and conclusions for the observability of the mathematical model can be derived from equations (28) to (30) [29].

An implicit integration method is used for the solution of the differential equations of state (23) [3, 18, 28].

3. The Method of Characteristics

4.1. The Basics

The method of characteristics developed in 1859 by Bernhard Riemann [30] has been used successfully since the 40ies of the 20th century for the solution of hyperbolic partial differential equations [31 - 40].

To derive the characteristic equations of an isothermal duct flow the system of equations (13) is written as follows:

$$\frac{\partial\ p}{\partial\ t} + c^2\ \frac{\partial\ \dot{m}}{\partial\ x} = 0\ ; \qquad (27)$$

$$\frac{\partial\ \dot{m}}{\partial\ t} + \frac{\partial\ p}{\partial\ x} = \ \varphi\ \left(p, \dot{m}\right)\ , \qquad (28)$$

with

$$\varphi\ \left(p, \dot{m}\right) = -\frac{\lambda \cdot c^2}{2 \cdot \gamma \cdot D} \cdot \frac{\dot{m} \cdot \left|\dot{m}\right|}{p}\ . \qquad (29)$$

The multiplication of equations (28) with sound velocity **c** and subsequent addition and subtraction with equation (27) yields the system of equations (27), (28) in its characteristic form:

$$\begin{aligned} &\left(\frac{\partial\ p}{\partial\ t} + c\ \frac{\partial\ p}{\partial\ x}\right) + c\ \left(\frac{\partial\ \dot{m}}{\partial\ t} + c\ \frac{\partial\ \dot{m}}{\partial\ x}\right) = c\ \varphi\ \left(p, \dot{m}\right) \\ &\left(\frac{\partial\ p}{\partial\ t} - c\ \frac{\partial\ p}{\partial\ x}\right) - c\ \left(\frac{\partial\ \dot{m}}{\partial\ t} - c\ \frac{\partial\ \dot{m}}{\partial\ x}\right) = -c\ \varphi\ \left(p, \dot{m}\right)\ . \end{aligned} \qquad (30)$$

Figure 2 shows in the x, t plane two straight lines, which are determined by the following equations:

$$\begin{aligned} &\frac{d\ x}{d\ t} = c\ , \qquad x - c\ t = \xi_0\ ; \\ &\frac{d\ x}{d\ t} = -c\ , \qquad x + c\ t = \eta_0\ . \end{aligned} \qquad (31)$$

These lines are called the characteristics of the system of partial differential equations (27), (28). The expressions

$$\left(\frac{\partial}{\partial t}+c\ \frac{\partial}{\partial x}\right) \text{ and } \left(\frac{\partial}{\partial t}-c\ \frac{\partial}{\partial x}\right)\ ; \quad \textbf{(32)}$$

are the total differentials of gas dynamic parameters, which are calculated along the characteristics with positive and negative slopes. The relation

$$\frac{d}{d\ t}\left(p+c\,\dot{m}\right)=c\ \varphi\ \left(p,\dot{m}\right)\ ; \quad \textbf{(33)}$$

holds along the straight lines of the first group, whereas the following relation holds along the second group:

$$\frac{d}{d\ t}\left(p-c\,\dot{m}\right)=-c\ \varphi\ \left(p,\dot{m}\right)\ . \quad \textbf{(34)}$$

The quantities $\left(p+c\,\dot{m}\right)$ and $\left(p-c\,\dot{m}\right)$ are termed the Riemann invariants. For the case $\varphi\ \left(p,\dot{m}\right)=0$, i.e., the friction term is neglected, they are constant along the lines

$$\frac{d\ x}{d\ t}=c \quad \text{und} \quad \frac{d\ x}{d\ t}=-c$$

In accordance with the above, the relation

$$p+c\,\dot{m}=const\,; \quad \textbf{(35)}$$

holds along the line $\overline{AM}$ (see Figure 2), and the relation

$$p-c\,\dot{m}=const \quad \textbf{(36)}$$

is valid along the line $\overline{BM}$.

Since the values of the pressure p and the mass flow rate $\dot{m}$ are known at t = 0, the values of $p(x,t)$ and $\dot{m}(x,t)$ can be computed from the well-known d'Alembert formulas [32]:

$$p_M(x,t) = \frac{p_0(x - c \cdot t) + p_0(x + c \cdot t)}{2} + c \cdot \frac{\dot{m}_0(x - c \cdot t) - \dot{m}_0(x + c \cdot t)}{2}; \quad \textbf{(37)}$$

$$\dot{m}_M(x,t) = \frac{p_0(x - c \cdot t) - p_0(x + c \cdot t)}{2 \cdot c} + \frac{\dot{m}_0(x - c \cdot t) + \dot{m}_0(x + c \cdot t)}{2}; \quad \textbf{(38)}$$

at any point **M** for any time plane. This method was used for the first time by *M. L. Bergeron* [41] for a graphical solution of the wave equation.

In the case when $\varphi\left(p,\dot{m}\right) \neq 0$ the d'Alembert formulas become inapplicable because the Riemann invariants are not constant along the characteristics. After the conversion of the differentials in equations (33) and (34) into finite differences they can be integrated approximately. For this purpose, the characteristics $x - c\ t = const$ and $x + c\ t = const$ are drawn in the x, t plane in the region $0 \leq x \leq L$ and $t \geq 0$, and they intersect at the (x_i, t_j) points (Figure 3).

The relations for the spatial and time steps then read:

$$\Delta x = \frac{L}{n}; \qquad x_i = i \cdot \Delta x \quad \text{for} \quad i = 0, 2, 3, \dots n$$

$$\textbf{(39)}$$

$$\Delta t = \frac{1}{c} \cdot \Delta x; \qquad t_j = j \cdot \Delta t \quad \text{for} \quad j = 0, 1, 2, 3, \dots$$

For stability reason, the Courant condition $\Delta\, t \le \dfrac{\Delta\, x}{c}$ is to be always satisfied [32].

Equations (37) and (38) in the form

$$\Delta(p + c \cdot \dot{m}) = c \cdot \varphi(p, \dot{m}) \cdot \Delta\, t = \varphi(p, \dot{m}) \cdot \Delta\, x$$

$$\Delta(p - c \cdot \dot{m}) = -c \cdot \varphi(p, \dot{m}) \cdot \Delta\, t = -\varphi(p, \dot{m}) \cdot \Delta\, x\,, \qquad (40)$$

imply according to Figure 3 the following recursion formulas for the computation of the unknown function values for p_i^j and $\dot{m}_i^j$ in the time plane t_j from the known values of these functions in the time plane t_{j-1}:

$$p_i^j = \frac{p_{i-1}^{j-1} + p_{i+1}^{j-1}}{2} + c \cdot \frac{\dot{m}_{i-1}^{j-1} - \dot{m}_{i+1}^{j-1}}{2} + \frac{\varphi_{i-1}^{j-1} - \varphi_{i+1}^{j-1}}{2}\, \Delta\, x\,; \qquad (41)$$

$$\dot{m}_i^j = \frac{p_{i-1}^{j-1} - p_{i+1}^{j-1}}{2 \cdot c} + \frac{\dot{m}_{i-1}^{j-1} + \dot{m}_{i+1}^{j-1}}{2} + \frac{\varphi_{i-1}^{j-1} + \varphi_{i+1}^{j-1}}{2 \cdot c}\, \Delta\, x\,, \qquad (42)$$

where the quantities φ_{i-1}^{j-1} and φ_{i+1}^{j-1} are determined from equation (29).

The values of the desired functions p_i^j and $\dot{m}_i^j$ can be determined with the aid of the recursion formulas (41) and (42) only in the region $0 < x < L$ and $t > 0$. For the boundary conditions $p(0,t)$, $p(L,t)$ or $\dot{m}(0,t)$, $\dot{m}(L,t)$ as well as for two initial conditions $p(x,0)$ and $\dot{m}(x,0)$, the corresponding recursion formulas must also be derived depending on the task to be solved.

Example:

$p_0^j = f_1(\dot{m}_0^j)$ - for the pressure at the inlet of pipeline (x = 0)

$\dot{m}_0^j = f_2(p_0^j)$ - for the mass flow rate at the inlet of pipeline (x = 0)

this implies for all $j \geq 1$

$$p_0^j - c \cdot \dot{m}_0^j = p_1^{j-1} - c \cdot \dot{m}_1^{j-1} - \varphi_1^{j-1} \cdot \Delta x; \quad (43)$$

$p_n^j = f_3(\dot{m}_n^j)$ - for the pressure at the outlet of pipeline (x = L)

$\dot{m}_n^j = f_4(p_n^j)$ - for the mass flow rate at the outlet of the pipeline (x = L)

this implies for all $j \geq 1$

$$p_n^j + c \cdot \dot{m}_n^j = p_{n-1}^{j-1} + c \cdot \dot{m}_{n-1}^{j-1} + \varphi_{n-1}^{j-1} \cdot \Delta x; \quad (44)$$

and for the case of a stationary initial condition

$$p_i^0 = \sqrt{p_0^2 - \frac{\lambda \cdot c^2 \cdot \dot{m}_0^2}{\gamma \cdot D} \cdot x_i}\,. \quad (45)$$

The derivation of the complete set of recursion formulas for the boundary and initial conditions (compressors, regulators, couplers, bypass, change in the nominal diameter, etc.) emerging in practical operation of networks would explode the size of the present contribution. Therefore, we have to refuse their presentation here.

4.2. Computation of Highly Dynamical Flow Processes

Highly dynamical (nonstationary) flow processes cannot be computed with the aid of the system of equations (13) or can be computed only in a rough approximation.

The method of characteristics is nevertheless very well appropriate for this purpose. Figure 4 shows the solution found with the aid of equations (41) and (42) for the case of blowing-out of a gas pipeline DN 200 10,000 m in length.

The boundary and initial conditions read:

$\dot{m}_0^j = 0$ - the mass flow rate at the pipeline inlet, that is at x = 0, is zero;

$p_n^j = p_a = 10^5\,Pa$ - the pressure at the pipeline end, that is at x = L, is equal to the air pressure

At t = 0 the pressure in pipeline is constant: $p_i^0 = 2 \cdot 10^5\,Pa$, i.e., the mass flow rate $\dot{m}_i^0 = 0$ is equal to zero ($\omega = 0$).

At the moment of time t = 0 the gas pipeline is suddenly opened at its end (x = L).

Under the neglection of the boundary and contraction effects of the outflowing gas, the pressure and mass flow rate presented in Figure 4 are obtained in the first 40 seconds.

The figure shows clearly a shock wave propagating against the flow direction, which, however, decays very rapidly in compressible media.

4.3. Computation of Non-Isothermal Flow Processes

The method of characteristics enables one to compute rapidly, simply, and with a high accuracy on the basis of the system of equations

$$\frac{\partial p}{\partial t} - \frac{p}{T} \cdot \frac{\partial T}{\partial t} + \frac{c^2}{\gamma} \frac{\partial \dot{m}}{\partial x} = 0 \; ; \tag{46}$$

$$\frac{\partial \dot{m}}{\partial t} + \frac{\partial p}{\partial x} = \varphi\left(p, \dot{m}, T \right); \tag{47}$$

$$\rho \cdot c_p \frac{\partial T}{\partial t} + \dot{m} \cdot c_p \frac{\partial T}{\partial x} - \frac{\partial p}{\partial t} = \Theta\, T \quad ; \tag{48}$$

$$\varphi\left(p, \dot{m}, T \right) = -\frac{\lambda \cdot c^2(T)}{2 \cdot \gamma \cdot D} \cdot \frac{\dot{m}\left|\dot{m}\right|}{p} \quad , \qquad \Theta\; T = \frac{4 \cdot q_n}{D} \; ; \tag{49}$$

$$p = \rho\, R\, T \; , \qquad c^2 = \gamma \cdot R \cdot T \; , \qquad \gamma = \frac{c_p}{c_v} \; . \tag{50}$$

also intrinsically non-isothermal flows, for example, the compression and rarefaction processes or the flows at a direct heat supply and removal, without the linearization of the partial differential equations (46) – (48) in a coupled form.

In the case of the above presented equations, one deals with a system of partial differential equations of hyperbolic type, which possess three real characteristics passing through each point of the x, t plane.

To determine these characteristics the lines are sought for in the x, t plane on which the ordinary differential equations exist, which involve the values of the functions $\boldsymbol{p}$, $\dot{\boldsymbol{m}}$ and $\boldsymbol{T}$ that are to be found.

If one sets **x = x (τ)** and **t = t (τ)**, where τ is aparameter along these lines, then the following relations are valid along these lines:

$$\frac{\partial p}{\partial x}\cdot\frac{d x}{d \tau}+\frac{\partial p}{\partial t}\cdot\frac{d t}{d \tau}=\frac{d p}{d \tau}; \qquad (51)$$

$$\frac{\partial \dot{m}}{\partial x}\cdot\frac{d x}{d \tau}+\frac{\partial \dot{m}}{\partial t}\cdot\frac{d t}{d \tau}=\frac{d \dot{m}}{d \tau}; \qquad (52)$$

$$\frac{\partial T}{\partial x}\cdot\frac{d x}{d \tau}+\frac{\partial T}{\partial t}\cdot\frac{d t}{d \tau}=\frac{d T}{d \tau}; \qquad (53)$$

These expressions associate the partial derivatives of **x** and **t** with the functions $\boldsymbol{p}$, $\dot{\boldsymbol{m}}$ and $\boldsymbol{T}$. Together with equations (46) – (48) they form a system of six algebraic equations for the determination of partial derivatives

$$\frac{\partial p}{\partial t},\quad \frac{\partial p}{\partial x},\quad \frac{\partial \dot{m}}{\partial t},\quad \frac{\partial \dot{m}}{\partial x},\quad \frac{\partial T}{\partial t},\quad \frac{\partial T}{\partial x}; \qquad (54)$$

of the functions which are to be determined.

It is here of interest only the case, in which the system of six equations has a variety of solutions. The „necessary condition“ for this is

$$\begin{pmatrix} 1 & 0 & 0 & c^2/\gamma & -p/T & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & c_p\rho & c_pT \\ \frac{dt}{d\tau} & \frac{dx}{d\tau} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{dt}{d\tau} & \frac{dx}{d\tau} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{dt}{d\tau} & \frac{dx}{d\tau} \end{pmatrix} = 0\,. \qquad (55)$$

Equation (55) implies (all terms of the order ≥ ω^2/c^2 are neglected)

the direction of characteristic 1 $\qquad \tau_1 : \quad \dfrac{dx_1}{dt} = c + \dfrac{\gamma-1}{2}\cdot\omega\,;$

the direction of characteristic 2 $\qquad \tau_2 : \quad \dfrac{dx_2}{dt} = -c + \dfrac{\gamma-1}{2}\cdot\omega \qquad (56)$

and the direction of characteristic 3 $\qquad \tau_3 : \quad \dfrac{dx_3}{dt} = \omega\,.$

Equations (56) cannot be integrated directly as in the case of an isothermal flow because they involve the functions, which depend on the desired solution:

$$c\,(T) = c\,(x,t) \text{ and } \omega = \omega\,(x,t)\,. \qquad (57)$$

They show, however, clearly that three curved characteristics pass through each point in the x, t plane.

Condition (55) dos not suffice for the solution of the above formulated problem. Along with the „necessary condition“, the following „sufficient condition“ must also be determined, which is termed in the profile literature also as „compatibility condition“ 35, 36]:

$$\begin{pmatrix} 1 & 0 & 0 & c^2/\gamma & -p/T & 0 \\ 0 & 1 & 1 & 0 & 0 & \varphi \\ -1 & 0 & 0 & 0 & c_p\rho & \Theta \\ \frac{dt}{d\tau} & \frac{dx}{d\tau} & 0 & 0 & 0 & \frac{dp}{d\tau} \\ 0 & 0 & \frac{dt}{d\tau} & \frac{dx}{d\tau} & 0 & \frac{d\dot{m}}{d\tau} \\ 0 & 0 & 0 & 0 & \frac{dt}{d\tau} & \frac{dT}{d\tau} \end{pmatrix} = 0. \qquad (58)$$

The soluition is as follows:

$$c_v \cdot \rho \cdot \gamma \cdot \left[\frac{\left(\frac{dx}{dt}\right)^2}{c^2} - 1 \right] \cdot \frac{\partial T}{\partial t} + \frac{d\dot{m}}{dt} \cdot \frac{dx}{dt} + \frac{\partial p}{\partial t} = \varphi \cdot \frac{dx}{dt} + \Theta \cdot \left(\frac{\gamma}{c^2} \cdot \left(\frac{dx}{dt}\right)^2 - 1 \right); \qquad (59)$$

Equation (59) forms together with the characteristics (56) the basis for the computation of one-dimensional non-isothermal nonstationary flows of compressible media.

If the temperature in the pipeline neighborhood does not depend on time then the heat flux through the pipe wall into or out of the surrounding ground can be computed with the aid of the formula [41]

$$\Theta = -\alpha_T \cdot (T - T_U)\ . \qquad (60)$$

The heat transfer coefficient α_T depends on the heat exchange between the gas and pipe wall, the heat conduction through the pipe wall including its coating, and on the heat exchange with the ground. A number of methods and formulas are known for its determination [17,42,43].

The characteristics (56) with condition (59) cannot be resolved in closed form. They must, therefore, be intgrated numerically, i.e., the differentials are replaced in the case of isothermal conditions by finite differences. In the case of the system of equations for the determination of the functions $p(x,t)$, $\dot{m}(x,t)$ and $T(x,t)$, the following formulas serve this purpose (Figure 5):

in the direction of the characteristic 1 - 3:

$$x - x_1 = \tau_{1,1}(t - t_1) \;, \tag{61}$$

in the direction of the characteristic 2 - 3:

$$x - x_2 = \tau_{2,2}(t - t_2) \;, \tag{62}$$

and in the direction of the characteristic 5 - 3:

$$x - x_3 = \tau_{3,5}(t - t_3) \;. \tag{63}$$

On each of these characteristics, the corresponding compatibility condition must be satisfied in accordance with equation (59). This finally gives also a system of equations for the determination of p, $\dot{m}$ and T at point 3. The computational diagram is presented in Figure 6.

The advantages of the method of characteristics are obvious. Figure 7 shows only for the demonstration purpose the pressure equalization between two 2,000 m long gas pipelines DN200, which are interconnected via a rapidly closing control valve. The natural gas under the pressure of 15 bar fills pipe I, and the gas in pipe II has the pressure below 10 bar. When the control valve is suddenly opened at time t = 0 at location x = 0, then this leads, as expected, to a cooling (rarefaction), and to a gas heating (compression) in pipe II. Depending on the input parameters, an equilibrium is established between the temperature gradient and the flow velocity. The consequence is that the contact zone between the cold and warm gas moves with a more or less significant velocity into pipe II. In the example under consideration, it has moved at a distance of only 100 m from the control valve after 60 seconds (Figure 7).

The example shows very clearly a close relation and the mutual effect of the pressure, flux, and temperature on the local flow conditions in high-pressure gas pipelines.

An adaptive simulation of flow processes in gas pressure regulator devices with preliminary heating is possible on the basis of the above numerical method. In this way, a preliminary heating adapted to the specific conditions (i.e., the compensation of the Joule-Thomson effect adapted to actual conditions) can be implemented on the basis of actually appearing temperature diminution. Careful computations show that this application of the method of characteristics oriented toward practice enables one to save annually over 2 Mio. DM on the power costs in the German gas economy.

Further application possibilities, which can only be mentioned here, are the design of a dynamic control of gas properties and the development of new tools for theoretical clarification of certain phenomena of retrograde condensation of the natural gases in the high-pressure networks.

Also when the three characteristics are in this case no more linear but curved, and cannot be determined explicitly from their ordinary differential equations, the advantages of the method of characteristics are immense.

In comparison with the classical difference method,

- The mathematical model of the method of characteristics describes and reflects remarkably the physical behaviour, which specifies the nonstationary flow through propagation, reflection, and superposition of pressure waves. The existence of these characteristics (also termed the Mach lines) has been proved long ago not only theoretically [30, 32, 36] but also practically [44].

- the method of characteristics is not „blind“, i.e., the modelled region agrees with the one, in which the processes to be descibed actually occur;

- the systems of partial hyperbolic differential equations are converted to the systems of equivalent algebraic equations or ordinary differential equations, which can easily be solved explicitly or implicitly;

- the spatial and time steps are physically related to each other in this solution method, which ensures a considerable reduction of the numerical dispersion.
- The method of characteristics is a better, more accurate and reliable model. Even highly dynamical and complex flow processes (the filling and …processes in gas pipelines, simulation of the effects at leekage, control of gas temperature and other gas properties, retrograde condensation, dust transport, etc.) can be modelled and computed with this method. The linearization of friction term is not needed.
-

Despite these outstanding properties, there are in the world considerable clauses for the use of the method of characteristics [3, 18, 45, 46]. There is a widespread opinion that the stability condition

$$\Delta t \leq \frac{1}{c} \Delta x \; ; \qquad \textbf{(64)}$$

represents a „significant limitation“ and, therefore, the method of characteristics does not suit for the mathematical modelling of nonstationary pressure, flow, and temperature variations in the gas networks.

In view of a high computing speed of the present-day personal computers, it is especially difficult to follow the widespread opinion that the input of small time steps (e.g. $\Delta t = 1 \ldots 3$ s) witnesses against the use of the method of characteristics. Even the computations over 24 hours with 3600 *24 $\approx$ 87,000 time steps represent no problem today for the advanced Pentium computers.

In addition, there is a widespread opinion that the deviation of the curved characteristic grid from the model grid may lead to errors under the consideration of temperature (Figure 8).

The gas flow velocity in high-pressure networks lies in subsonic range, i.e., at relatively small Mach numbers. This implies:

$$(w / c)^2 << w / c$$

(65)

$$\frac{w}{c} \approx \frac{1}{40} \approx 0,025 \qquad \frac{d\ x_{1,2}}{d\ t} = \pm c \ .$$

This means that the deviation of the curved characteristic grid from the model grid lies within the limits of the numerical acuracy. In addition, a correct choice of the equation for heat exchange (Θ) with the environment is so problematic for nonstationary processes that all the remaining qustions about a „more or less high accuracy“ get completely into the background.

4. Conclusion

Basing on a brief overview of the developments in the field of gas network simulation, its theoretical background and available practical experiences at the design of simulation accompanying the flow processes in a regional high-pressure network, the author has presented the possibilities for the use of the method of characteristics as an innovative supplement to already available models (GANESI, SIMONE) verified in practice.

The first cautious computations show that only in the field of preliminary gas heating via a process oriented heat supply on the basis of an adaptive model of actually appearing temperature diminution in the gas pressure control devices (that is a compensation of the Joule - Thomson effect adapted to actual conditions) with the aid of the method of characteristics, over 2 million DM can be saved annually in the German economy on the power costs.

Besides the application for the temperature control in gas networks, the method of characteristics opens up quite new prospects for the design of calibrated control of the gas properties as well as for theoretical clarification of certain phenomena of the retrograde condensation of natural gases in the high-pressure networks.

* Devoted to the outstanding Russian academician, the teacher and sincere friend, Prof. Michail Vladimirovitsch Lurje, head of the chair for fluid dynamics of the Russian State Gubkin University for Petroleum and Gas in Moscow, in connection with his 60th anniversary.

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